

Interjet Rapidity Gaps in Perturbative QCD[†]

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Abstract

We discuss a formalism in which high- p_T dijet rapidity gaps are identified by energy flow in the interjet region. When the gap energy, Q_{gap} , is sufficiently large, the cross section may be computed from standard factorization theorems. For $p_T \gg Q_T \gg \Lambda_{\text{QCD}}$, this is a two-scale perturbative problem, in which we may resum logarithms of Q_{gap}/p_T . The cross section computed as a function of Q_{gap} reproduces many of the features of the Tevatron dijet gap data. The factorized cross section gives meaning to the color content of the hard scattering.

1 Dijet Gaps and Color

One of the enduring themes of high energy physics is scattering through the exchange of composite systems. An exchange between incoming hadrons that produces two sets of outgoing hadrons separated by a large gap in rapidity is necessarily color singlet. Perturbation theory becomes relevant when one end of the exchange involves a hard scattering, producing jets or heavy quarks. In this talk, we discuss the application of perturbative methods to exchanges that are hard at *both* ends, and in which high- p_T jets are observed at large rapidity in both the forward and backward directions, with little or no particle multiplicity between. These are often called “dijet rapidity gap” events.

As a heuristic principle, based in part on the antenna patterns of QED, we expect the exchange of gluons in a color singlet state to produce very little radiation in the intermediate rapidity region [1]. This idea has such appeal that dijet gap events are routinely termed “color singlet exchange”. To the extent that it is not a truism, this term is a description of the color of gluons exchanged at *short* distances and times. The idea of color singlet exchange, however, has been a difficult one to implement in perturbative terms. After all, gluons of any energy carry octet color charge, so that there is no unique way of defining color exchange in a finite amount of time [2]. On the other hand, it takes a very short time to radiate a hard gluon, and once radiated, it cannot be reabsorbed on the basis of soft color rearrangements at very long times.

2 The Two-Gluon Model, Soft Color and Factorization

The simplest short-distance model for dijet gaps is based on two-gluon exchange [1]. In a two-gluon model, the gap is usually filled by spectator interactions, up to a “survival

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probability", P_S , which may be estimated [1, 3] from low- p_T diffractive scattering to be of order 0.1. Denoting the probability for hard color-singlet exchange as f_1 , the fraction of gap events becomes

$$f_{\text{gap}} = f_1 P_S. \quad (1)$$

If we estimate $f_1 \sim \mathcal{O}(\alpha_s(p_T)/\pi) \sim 0.1$, we predict gap events at the one percent level, and this is what is seen experimentally [4, 5, 6, 7]. This analysis would lead us to expect more gap events from gluon-gluon than quark-quark scattering, because of the larger color factors in two-gluon exchange graphs for the former. This expectation was tested by comparing 630 and 1800 GeV data from the Tevatron, because at fixed p_T the role of gluon-gluon scattering increases with the overall center-of-mass energy. The proportion of gap events, however, decreased, rather than increased, with the energy.

In the soft color approach [8], normally presented as an alternative to the two-gluon model, the underlying hard scattering is treated at lowest order, which for gap events is primarily single-gluon, color octet exchange. The gap probability is determined by counting possible color exchanges, assuming all to be equally likely, up to an overall survival factor (rather larger than 1/10). Because gluons have more color states than quarks, they are correspondingly less likely to produce gap events. The soft color model then naturally leads to fewer gap events as the energy, and hence the role of gluons, increases.

Clearly, dijet gaps provide insight into a tantalizing mixture of short- and long-distance physics. The successes of the soft color picture suggest, however, that to learn more about color exchange at short distances, we must generalize the two-gluon model. From the point of view of perturbative QCD this is already necessary, because two-gluon exchange is not, by itself, infrared safe. That is, even at lowest order it is sensitive to long-distance configurations where one gluon carries almost all the momentum transfer, and the other almost none. It is therefore natural to seek a definition of color exchange that is infrared safe, and hence well-defined in perturbation theory. To do so, we find it useful to extend the concept of gap events.

We have developed a (resummed) perturbative QCD formalism for dijet rapidity gaps, made possible by redefining gaps in terms of an energy flow, Q_{gap} , rather than particle multiplicity [9]. The resulting cross sections can be treated via standard factorization theorems. In this formulation, if $Q_{\text{gap}} \gg \Lambda_{\text{QCD}}$ the cross section is perturbatively calculable. In addition, when $p_T \gg Q_{\text{gap}} \gg \Lambda_{\text{QCD}}$, our gap cross sections have two perturbative scales, and logarithms in their ratio can be resummed by renormalization group methods.

Resummation in $\ln(Q_{\text{gap}}/p_T)$ allows us to probe color flow at short distances, and to generalize the concept of hard color singlet exchange. As we shall see, for Tevatron kinematics, the dominant short-distance color exchange in rapidity gap events has a healthy proportion of color octet.

The dijet cross section at measured $Q_{\text{gap}} \gg \Lambda_{\text{QCD}}$ falls into the class of inclusive jet cross sections that can be written in factorized form:

$$\begin{aligned} \frac{d\sigma}{dQ_{\text{gap}} d\cos\hat{\theta}}(S, E_T, \Delta y) &= \sum_{f_A, f_B} \int d\cos\hat{\theta} \int_0^1 dx_A \int_0^1 dx_B \phi_{f_A/p}(x_A, -\hat{t}) \phi_{f_B/\bar{p}}(x_B, -\hat{t}) \\ &\times \sum_{f_C, f_D} \frac{d\hat{\sigma}^{(f)}}{dQ_{\text{gap}} d\cos\hat{\theta}}(\hat{t}, \hat{s}, y_{JJ}, \Delta y, \alpha_s(\hat{t})), \end{aligned} \quad (2)$$

with $\phi_{f/h}$ parton distributions, evaluated at scale $\sqrt{-\hat{t}}$, the dijet momentum transfer. The partonic cross section $d\hat{\sigma}^{(f)}/dQ_{\text{gap}} d\cos\hat{\theta}$ is a hard scattering function, starting with the Born cross section at lowest order (cf. the soft color model). The index f denotes the

partonic hard scattering $f_A + f_B \rightarrow f_C + f_D$. The cross section depends on the dijet pair rapidity y_{JJ} , the partonic center-of-mass (c.m.) energy squared \hat{s} , the partonic c.m. scattering angle $\hat{\theta}$, with $-\frac{\hat{s}}{2}(1 - \cos \hat{\theta}) = \hat{t}$, and Δy , the gap size as a rapidity interval.

3 Refactorization and Resummation

Our central observation is that we may further “refactorize” the hard-scattering cross section in Eq. (2) to separate the gap energy dependence, at scale $Q_{\text{gap}} \gg \Lambda_{\text{QCD}}$, from the underlying hard scattering, at scale $p_T \gg Q_{\text{gap}}$. This factorization depends upon the color exchange in the residual hard scattering. Taking into account the hard-scattering amplitude and its complex conjugate, we have the following matrix relation for $\hat{\sigma}$ in (2):

$$Q_{\text{gap}} \frac{d\hat{\sigma}^{(\text{f})}}{dQ_{\text{gap}} d\cos \hat{\theta}} (\hat{s}, \hat{t}, y_{JJ}, \Delta y, \alpha_s(-\hat{t})) = H_{IL} \left(\frac{\sqrt{-\hat{t}}}{\mu}, \sqrt{\hat{s}}, \sqrt{-\hat{t}}, \alpha_s(\mu^2) \right) \times S_{LI} \left(\frac{Q_{\text{gap}}}{\mu}, y_{JJ}, \Delta y \right), \quad (3)$$

where H_{IL} incorporates scattering at scale p_T , and S_{LI} the intermediate radiation scale Q_{gap} . Because our cross section is inclusive, dependence on momentum scales below Q_{gap} cancels. The parameter μ is a new “refactorization scale”, which must be introduced to define the tensors H_{IL} and S_{LI} . Their indices refer to a basis of color exchanges in the amplitude and the complex conjugate at the shortest distance scale (interference is possible). For example, in quark-antiquark scattering, we may choose the basis: $I, J = t$ -channel singlet, t -channel octet.

The left-hand side of the refactorization relation Eq. (3) is independent of the new factorization scale μ , a requirement that immediately leads to renormalization group equations, [10]

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right) S_{LI} = -(\Gamma_S^\dagger)_{LB} S_{BI} - S_{LA} (\Gamma_S)_{AI}, \quad (4)$$

for S , and similarly for H , where $\Gamma_S(\alpha_s)$ is an anomalous dimension matrix that has been computed at one loop [9, 11]. Thus, the color content of the hard scattering H_{IL} determines the Q_{gap} -dependence of the cross section, through the eigenvalues of Γ_S , which themselves depend on the flavors and scattering angles of the underlying hard scattering. To leading logarithm in $Q_{\text{gap}}/\sqrt{-\hat{t}}$, we find

$$\begin{aligned} \frac{d\hat{\sigma}^{(\text{f})}}{dQ_{\text{gap}} d\cos \hat{\theta}} (\hat{s}, \hat{t}, y_{JJ}, \Delta y, \alpha_s(-\hat{t})) &= \\ H_{\beta\gamma}^{(1)} \left(\Delta y, \sqrt{\hat{s}}, \sqrt{-\hat{t}}, \alpha_s(-\hat{t}) \right) S_{\gamma\beta}^{(0)}(\Delta y) & \\ \times \frac{E_{\gamma\beta}}{Q_{\text{gap}}} \left[\ln \left(\frac{Q_{\text{gap}}}{\Lambda} \right) \right]^{E_{\gamma\beta}-1} \left[\ln \left(\frac{\sqrt{-\hat{t}}}{\Lambda} \right) \right]^{-E_{\gamma\beta}}. & \end{aligned} \quad (5)$$

In this expression, written in the color tensor basis that diagonalizes the anomalous dimension matrix Γ , the exponents $E_{\gamma\beta}$ are given in terms of the eigenvalues, $\lambda_\beta = (\alpha_s/\pi)\lambda_\beta^{(1)}$, of Γ by

$$E_{\gamma\beta} (y_{JJ}, \hat{\theta}, \Delta y) = \frac{2}{\beta_0} \left[\lambda^{(1)*}_\gamma (y_{JJ}, \hat{\theta}, \Delta y) + \hat{\lambda}_\beta^{(1)} (y_{JJ}, \hat{\theta}, \Delta y) \right], \quad (6)$$

with $\beta_0 = 11N_c/3 - 2n_f/3$. The linear combination of color exchanges with smallest eigenvalue thus dominates the behavior of the cross section in the limit $Q_{\text{gap}}/p_T \rightarrow 0$.

The concept of a dominant eigenvalue generalizes conventional hard singlet exchange, because the eigenvectors of the anomalous dimension matrix are linear combinations of elements in the basis of t -channel color transfers. The coefficients depend, in general, on the scattering angle of the hard scattering. Eq. (6) thus leads to a detailed set of predictions for dijet data with measured interjet energy flow.

4 Rapidity and Gap Energy Dependence

Explicit forms of the anomalous dimension eigenvalues λ_α for quark and gluon processes may be found in Ref. [11]. Here, we would like only to emphasize a few general features. First, the overlap of the dominant eigenvector with hard color singlet exchange grows in the direction of forward scattering, so that in the Regge limit $-\hat{t}/\hat{s} \rightarrow 0$, \hat{t} fixed, the dominant color exchange becomes purely color singlet [12]. Second, the eigenvalues for gluon-gluon scattering are larger than those for processes involving quarks. This makes it harder for gluon-gluon hard scattering to produce rapidity gaps, for much the same reasons as in the soft color model. The size of the eigenvalue λ_α is related to the number of color states available.

A typical prediction is shown in Fig. 1 [9], where we illustrate the differential cross section for measured Q_{gap} at 630 and 1800 GeV, the former with a gap of 3.2 units of rapidity, and latter at 4 units, following the D0 experiment. These curves were generated from valence quarks and antiquarks only, so at any scattering angle color exchange is a linear combination of singlet and octet. For any of the scattering angles allowed by the kinematics, the smallest (“dominant”) eigenvalue as $Q_{\text{gap}} \rightarrow 0$ is primarily, but not completely, singlet, and the other primarily octet. We refer to them “quasi-singlet” and “quasi-octet”, respectively. The projection of the hard scattering on the t -channel eigenvectors is nonvanishing for both quasi-singlet and quasi-octet, but is larger for quasi-octet. Examining Eq. (6), we see that the Q_{gap} -dependence of the cross section for each color exchange is determined by the size of the eigenvalues, along with the projection.

The quasi-singlet contribution, shown by dashed lines in the figure, has an eigenvalue that is less than unity in absolute value. For each value of $-\hat{t}$ it is therefore a monotonically decreasing function of Q_{gap} . The differential cross section has a weak singularity at $Q_{\text{gap}} = \Lambda_{\text{QCD}}$, but its moments are calculable. Quasi-octet exchange (dotted lines) on the other hand, has an eigenvalue that is always greater than unity. For small $Q_{\text{gap}} > \Lambda_{\text{QCD}}$, it therefore increases in Q_{gap} until the dimensional factor $1/Q_c$ takes over. The sum of these two contributions, along with their nonvanishing interference, is shown by the solid lines in the figure. For most of the range in Q_{gap} , it has the same shape as the quasi-octet curve: small for small Q_{gap} , reaching a peak at order $\sqrt{-\hat{t}}$, and then decreasing. At low gap energy, however, the quasi-singlet exchange produces a small upturn. This is the “hard singlet exchange” observed by CDF and D0 [4, 5]. Gap events defined by vanishing particle multiplicity in the interjet region are counted in this excess. Our prediction for such events must be diluted, as usual, by corrections associated with spectator interactions, which, according to the factorization formalism, are suppressed only by powers of $\Lambda_{\text{QCD}}/Q_{\text{gap}}$, and which therefore become important for small Q_{gap} . We have in Eq. (2), however, a set of predictions for the full range of Q_{gap} .

Recently, gluonic processes were considered as part of a complete analysis of dijet gaps in photoproduction, seen by Zeus [11]. As noted above, for gluon-gluon scattering the

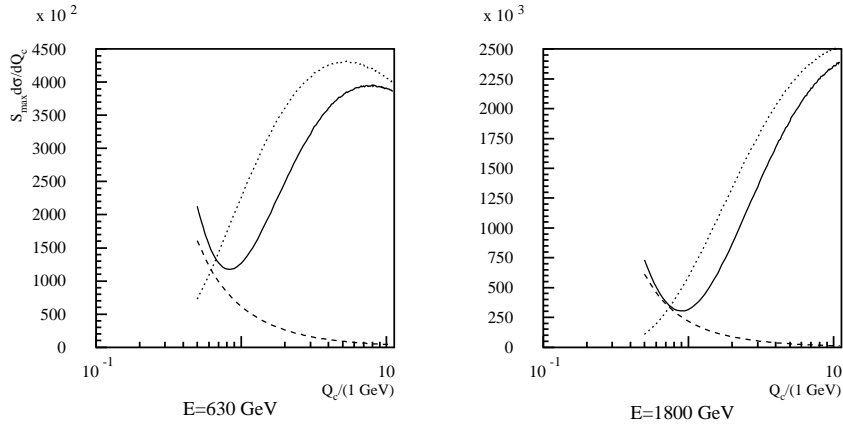


Figure 1: The cross section as a function of gap energy, labelled Q_c (solid line) and contributions from “quasi-octet” (dotted line) and quasi-singlet (dashed line) color exchange, for 630 and 1800 GeV. Units are arbitrary.

eigenvalues are larger than for quark-antiquark scattering, and, indeed, are greater than unity over the accessible phase space. As a result, gluonic processes lack the upturn we have just found at low Q_{gap} for quark processes. This is the explanation for the lower gap cross section observed as the role of gluons increases [6].

5 Summary

Energy flow analysis makes possible a quantitative study of radiation in interjet regions, and gives a perturbative meaning to short-distance color exchange, generalizing the two-gluon exchange model. On the basis of this analysis, gaps in dijet events come from a compound structure, predominantly, but not purely, singlet in the hard scattering. Many qualitative results, including the relative suppression of dijet gaps for gluon-gluon scattering, are similar to those of the soft color model. Although much more remains to be done, the perturbative analysis offers a systematic set of differential predictions for energy flow, as a function of momentum transfer, flavor and gap width. In principle, these ideas can be tested at Run II of the Tevatron, at Hera and at the LHC.

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